

Prospecting for the Superfluid Transition in Electron-Hole Coupled Quantum Wells Using Coulomb Drag

Ben Yu-Kuang Hu

Department of Physics, University of Akron, Akron, Ohio 44325-4001

(Received 2 August 1999)

I investigate the possibility of using Coulomb drag to detect a precursor of the predicted (but as yet not definitively observed) superfluid transition in electron-hole coupled quantum wells. The drag transresistivity ρ_{21} is shown to be significantly enhanced above the mean-field transition temperature T_c and to diverge logarithmically as $T \rightarrow T_c^+$, due to electron-hole pairing fluctuations which are somewhat analogous to the Maki-Thompson contribution to conductivity in metals above the superconducting T_c . The enhancement in ρ_{21} is estimated to be detectable at temperatures much higher than T_c .

PACS numbers: 73.50.Dn, 71.35.Lk

Under suitable conditions, an attractive interaction between fermions causes a pairing instability that leads to phase transition, such as superconductivity, where phonons mediate an effective attractive interaction between electrons. Since the direct Coulomb interaction is in general stronger than effective phonon-mediated interactions, a coupled electron-hole gas might be expected to readily undergo an equivalent phase transition. This possibility was theoretically investigated in the 1960s, and it was predicted that electron-hole systems in bulk systems would form a condensate at low enough temperatures [1]. Despite a formal similarity between electron-hole condensate and the BCS superconducting states, the former was shown to be, in fact, an insulator. Subsequently, it was proposed [2] that *separated* electron-hole systems could form a superfluid state, due to the suppression of transitions that fix the phase of the order parameter thereby producing an insulator. The separation of the electron and hole components in semiconductor systems also has an important experimental advantage; it inhibits the recombination of the electrons and holes which would otherwise rapidly deplete the system.

The advent of fabrication techniques such as molecular beam epitaxy has made it possible to produce high quality electron-hole double well systems and to test these predictions. Experimental searches for double layer electron-hole condensation have generally used optical techniques. While there are reports of optical signatures [3] hinting at a transition, there has been no definitive observation of a superfluid condensation. Recently, Vignale and MacDonald [4] proposed that the superfluid transition could be definitively identified through a completely different route: using Coulomb drag.

In Coulomb drag experiments, each well in the coupled well system is individually electrically contacted, making it possible to isolate a driving voltage in just one of the two wells. When an electrical current is driven through one well, the interlayer interactions produce a measurable drag signal in the other. The measured quantity is typically the transresistivity $\rho_{21} = E_2/J_1$, where J_1 is the current density in the driving layer and E_2 is the electric field response

in the other layer when it is in an open circuit. There has been considerable experimental [5] and theoretical [6] activity in drag in the past decade.

For most typical experimental parameters, the interlayer interaction is a weak perturbation, in which case drag is adequately described within the Born approximation; i.e., the carriers in the adjacent wells are uncorrelated, and carriers in one well occasionally collide with carriers in the other. Typically, ρ_{21} is much smaller than the resistivities of the individual layers, ρ_{11} and ρ_{22} . Furthermore, in this weak-coupling regime the Born approximation unambiguously predicts $\rho_{21} \rightarrow 0$ as the temperature T goes to zero, because the scattering phase space for the quasiparticles vanishes at zero temperature. What Vignale and MacDonald [4] showed is that when electron-hole condensation occurs, ρ_{21} becomes *comparable* in magnitude to ρ_{11} and ρ_{22} , and *diverges* when $T \rightarrow 0$. As yet, this has not been experimentally observed.

Even when the electrons and holes are not in a condensed state, interlayer correlations can affect the transresistivity [7]. In the closely related case of drag of composite fermions (electrons bound to two magnetic flux quanta), the buildup of interlayer correlations was shown theoretically to produce unique signatures in the transresistivity [8] which differ markedly from the weak-coupling result $\rho_{21}(T \rightarrow 0) = 0$. Deviations from the weak-coupling result in composite fermion drag at low temperatures have, in fact, been reported [9]. Analogously, deviations from the weak-coupling form of $\rho_{21}(T)$ in electron-hole systems would indicate the presence of interlayer correlations which foreshadow an electron-hole condensation; thus Coulomb drag can be used to “prospect” for situations where electron-hole condensation could occur.

In this paper, I evaluate the transresistivity in the normal state of an electron-hole double quantum well system, going beyond the Born approximation and taking into account correlations between the two wells. I use the Kubo formalism to calculate $\sigma_{21} = J_1/E_2$, from which the transresistivity is given by $\rho_{21} \approx -\sigma_{21}(\sigma_{11}\sigma_{22})^{-1}$. The Hamiltonian I use to describe this system is

$$H = \sum_{i=1,2} \sum_{\mathbf{k}_i} \epsilon_i(\mathbf{k}_i) \hat{c}_i^\dagger(\mathbf{k}_i) \hat{c}_i(\mathbf{k}_i) + \sum_{\mathbf{q}} \hat{\rho}_1(\mathbf{q}) \hat{\rho}_2(-\mathbf{q}) V(\mathbf{q}) + \hat{H}_{\text{imp}}. \quad (1)$$

Here $\hat{c}_i^{(\dagger)}$ are the particle field operators of layer $i = 1, 2$ (the drive and drag layers, respectively), ϵ_i is the kinetic energy, $\hat{\rho}_i$ is the particle density operator, and \hat{H}_{imp} is the contribution from static impurities, which are assumed to be uncorrelated [10]. ‘‘Particle’’ refers to either electron or hole. Within linear response, it does not matter which layer is driven. The static transconductivity $\sigma_{21} = -e_1 e_2 \lim_{\omega \rightarrow 0} \text{Im}[\Pi(\omega)/\omega]$ where e_i are the charges in layer i and $\Pi(i\Omega_n) = \mathcal{A}^{-1} \int_0^{T^{-1}} d\tau e^{i\Omega_n \tau} \langle \hat{j}_1(i\tau) \hat{j}_2(0) \rangle$ is the current-current correlation function (\mathcal{A} = sample area, Ω_n = Matsubara boson frequency). In this paper, $\hbar, k_B = 1$.

To evaluate σ_{21} , the standard diagrammatic techniques [11,12] are used. The terms I concentrate on are related to the Maki-Thompson terms used to study effects of superconducting fluctuations in the conductivity of metals above T_c , and the Feynman diagrams are shown in Figs. 1(a) and 1(b). Physically, these terms describe ‘‘the effect of ephemeral Cooper pairs on the conductivity’’ [13]. Composite fermion drag [8,9] differs from electron-hole drag in that the composite fermions have relatively large masses and the phase-breaking processes are dominant [14], which favor the Aslamazov-Larkin terms [13,15]. It is assumed in this paper that the samples are clean and phase-breaking processes are weak. The correlations are described by \mathcal{T}

$$\Pi(i\Omega_n) = -\frac{4T^2}{\mathcal{A}^2} \sum_{\mathbf{k}_1, \mathbf{k}_2} v_{1,x}(\mathbf{k}_1) v_{2,x}(\mathbf{k}_2) \sum_{ik_{1,n}} G_1(\mathbf{k}_1, ik_{1,n} + i\Omega_n) G_1(\mathbf{k}_1, ik_{1,n}) \sum_{ik_{2,n}} G_2(\mathbf{k}_2, ik_{2,n} + i\Omega_n) G_2(\mathbf{k}_2, ik_{2,n}) \times [\langle \mathbf{k}_{\text{pp}} | \mathcal{T}_{\text{pp}}(\mathbf{P}_{\text{pp}}; ik_{1,n} + ik_{2,n} + i\Omega_n) | \mathbf{k}_{\text{pp}} \rangle + \langle \mathbf{k}_{\text{pa}} | \mathcal{T}_{\text{pa}}(\mathbf{P}_{\text{pa}}; ik_{1,n} - ik_{2,n}) | \mathbf{k}_{\text{pa}} \rangle]. \quad (2)$$

Here, G_i is the particle Green function in layer i , and $k_{i,n}$ are Matsubara fermion frequencies. The particle velocities are given by $\mathbf{v}_i = \mathbf{k}_i/m_i$. For convenience, the momentum dependences of the \mathcal{T} matrices are parametrized by the center-of-mass coordinates, $\mathbf{P}_{\text{pp}} = \mathbf{k}_1 \pm \mathbf{k}_2$ and $\mathbf{k}_{\text{pp}} = x_2 \mathbf{k}_1 \mp x_1 \mathbf{k}_2$, where $x_i = m_i/(m_1 + m_2)$.

To obtain the static transresistivity, the analytic continuation $i\Omega_n \rightarrow \omega + i0^+$, followed by the limit $\omega \rightarrow 0$,

$$\sigma_{21} = -\frac{8\pi\tau_1\tau_2 e_1 e_2}{T} \int \frac{d\mathbf{k}_1}{(2\pi)^2} \int \frac{d\mathbf{k}_2}{(2\pi)^2} \int \frac{d\mathbf{q}}{(2\pi)^2} v_{1,x}(\mathbf{k}_1) v_{2,x}(\mathbf{k}_2) \{ n_F(\xi_{\mathbf{k}_1}) n_F(\xi_{\mathbf{k}_2}) n_F(-\xi_{\mathbf{k}_1+\mathbf{q}}) \times n_F(-\xi_{\mathbf{k}_2-\mathbf{q}}) | \langle \mathbf{k}_{\text{pp}} | \mathcal{T}_{\text{pp}}(\mathbf{P}_{\text{pp}}, \xi_{\mathbf{k}_1} + \xi_{\mathbf{k}_2}) | \mathbf{k}_{\text{pp}} + \mathbf{q} \rangle |^2 \delta(\xi_{\mathbf{k}_1} + \xi_{\mathbf{k}_2} - \xi_{\mathbf{k}_1+\mathbf{q}} - \xi_{\mathbf{k}_2-\mathbf{q}}) - n_F(\xi_{\mathbf{k}_1}) n_F(-\xi_{\mathbf{k}_2}) \times n_F(-\xi_{\mathbf{k}_1+\mathbf{q}}) n_F(\xi_{\mathbf{k}_2+\mathbf{q}}) | \langle \mathbf{k}_{\text{pa}} | \mathcal{T}_{\text{pa}}(\mathbf{P}_{\text{pa}}, \xi_{\mathbf{k}_1} - \xi_{\mathbf{k}_2}) | \mathbf{k}_{\text{pa}} + \mathbf{q} \rangle |^2 \delta(\xi_{\mathbf{k}_1} - \xi_{\mathbf{k}_2} - \{\xi_{\mathbf{k}_1+\mathbf{q}} - \xi_{\mathbf{k}_2+\mathbf{q}}\}) \}. \quad (3)$$

Here, $\xi_{\mathbf{k}_i} = \epsilon_i(\mathbf{k}_i) - \mu_i$, where μ_i is the chemical potential of layer i , and n_F is the Fermi function.

Equation (3) can to a certain extent be interpreted within the framework of the semiclassical Boltzmann equation [19]. The first terms within the integral of Eq. (3) correspond to the contribution of particle-particle

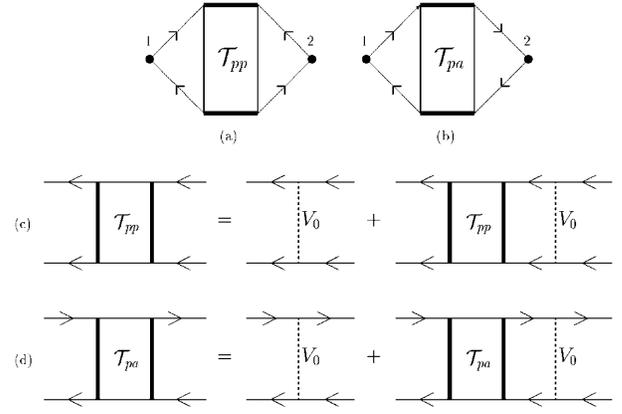


FIG. 1. Feynman diagrams used in calculating the transconductivity. (a) and (b) show the particle-particle and particle-antiparticle channel contributions to σ_{21} . The black dots are the current vertices, the numbers above them denote the layer indices, and the arrows are the particle Green functions. The \mathcal{T}_{pp} and \mathcal{T}_{pa} are the particle-particle and particle-antiparticle \mathcal{T} matrices which are calculated using the Bethe-Salpeter equation, shown diagrammatically in (c) and (d), respectively.

matrices which are given by the Bethe-Salpeter equation, shown in Figs. 1(c) and 1(d). \mathcal{T}_{pp} and \mathcal{T}_{pa} correspond to the particle-particle (pp) and the particle-antiparticle (pa) channels, respectively. (The term ‘‘antiparticle’’ rather than the standard ‘‘hole’’ is used to avoid the dual use of the word hole.)

If the interlayer interaction is assumed to be static, the frequency dependence of \mathcal{T}_{pp} (\mathcal{T}_{pa}) comes only from the sum (difference) of the energies in the vertices. Then, from standard diagrammatic rules [16],

must be taken. To simplify the expressions, I assume that both wells are weakly disordered (justified by the excellent quality of the two-dimensional electron and hole gases produced by molecular beam epitaxy), and hence only terms to lowest order in τ_i^{-1} , the inverse particle lifetime, are kept [17]. Using this approximation and the generalized optical theorem [18] to relate $\text{Im}[\mathcal{T}]$ to $|\mathcal{T}|^2$, I obtain after some algebra

scattering from state $\mathbf{k}_1, \mathbf{k}_2$ with exchange of momentum \mathbf{q} , when the distribution of layer 1 is shifted by the driving electric field, while the second corresponds to the contribution due to Pauli blocking of electrons going into state $\mathbf{k}_1 + \mathbf{q}$. Note that with a purely

semiclassical argument, it is impossible to have different scattering rates \mathcal{T}_{pp} for particle-particle and \mathcal{T}_{pa} for particle-antiparticle collisions, as is obtained above from a full many-body calculation. In the Born approximation limit, $\langle \mathbf{k} | \mathcal{T}_{pp}(\mathbf{P}, \omega + i0^+) | \mathbf{k} + \mathbf{q} \rangle = \langle \mathbf{k} | \mathcal{T}_{pa}(\mathbf{P}, \omega + i0^+) | \mathbf{k} + \mathbf{q} \rangle = V(q, \omega) \equiv$ the screened interlayer interaction, one regains the standard weak-coupling result [11,12,19,20] which typically involves a product of the individual layer susceptibilities (or, in the general case, a closely related function [11]). The above shows that when correlations between layers are included, the expression for σ_{21} cannot be written in the standard weak-coupling form.

When the \mathcal{T} matrix is regular (as with the case of hard-core fermions) the temperature dependence of the

transresistivity is not qualitatively different from the weak-coupling case, despite the presence of interlayer correlations. This result is consistent with the claim that a nonzero transresistivity at $T = 0$ is possible only with a phase transition [21]. When a phase transition such as superconductivity or electron-hole condensation takes place, the \mathcal{T} matrix acquires a singularity at the mean-field transition temperature T_c [22]. Below, I examine σ_{21} for temperatures well above T_c [23].

For simplicity, to obtain qualitative features let us assume a simple local interlayer interaction, $V(q) = V_0 < 0$. Then the \mathcal{T} matrices within the Bethe-Salpeter approximation are dependent only on the total momentum \mathbf{P} and energy ω of the incoming particles. The $\mathcal{T}_{pp}(\mathbf{P}, \omega)$ channel diverges at T_c , $\omega = 0$, and $\mathbf{P} = 0$. Its explicit form is

$$\mathcal{T}_{pp}(P, \omega; T) = -\frac{|V_0|}{1 + |V_0|\chi_{pp}(P, \omega; T)}, \quad (4a)$$

$$\chi_{pp}(P, \omega; T) = \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{1 - n_F(\xi_{1,x_1\mathbf{P}+\mathbf{k}}) - n_F(\xi_{2,x_2\mathbf{P}-\mathbf{k}})}{\omega - E_k - \mathcal{E}_P + \mu_1 + \mu_2 + i0^+}. \quad (4b)$$

Here, $E_k = k^2(m_1^{-1} + m_2^{-1})/2$ and $\mathcal{E}_P = P^2/2(m_1 + m_2)$.

The singularity in the \mathcal{T}_{pp} occurs when $\chi_{pp} = -|V_0|^{-1}$. This occurs at the highest temperature when $P = 0$, $\omega = 0$, and $k_{F,1} = k_{F,2} \equiv k_F$ (matched Fermi surfaces). Expanding $\chi_{pp}(T)$ about the zeros of P , ω , and $\Phi \equiv [k_{F,2}^2 - k_{F,1}^2]/2(m_1 + m_2)$, and substituting the resultant expression into Eq. (3) yields (assuming $\sigma_{21}^2 \ll \sigma_{11}\sigma_{22}$, and reintroducing \hbar)

$$\rho_{21} = -\frac{\hbar}{e_1 e_2} \frac{T^2}{E_{k_F}^2} \frac{8\pi^2}{7\zeta(3)} I(x_1, \tilde{\Phi}, s), \quad (5a)$$

$$I(x_1, \tilde{\Phi}, s) = \int_{-\infty}^{\infty} \frac{dy}{y} \frac{\tan^{-1}(y/s)}{\{\cosh(y/2) + \cosh[y(1/2 - x_1) + \tilde{\Phi}]\}^2}, \quad (5b)$$

where $s = 8\pi^{-1} \log(T/T_c) + 14\zeta(3)\pi^{-3}\tilde{\Phi}^2$ and $\tilde{\Phi} \equiv \Phi/T_c$ [24].

For matched electron and hole densities (i.e., $\Phi = 0$),

$$I[x_1, 0, s = 8\pi^{-1} \log(T/T_c)] \approx \begin{cases} 1/s & \text{if } s \gg 1, \\ \pi \ln(1/s) & \text{if } s \ll 1, \end{cases} \quad (6)$$

ignoring prefactors which depend weakly on x_1 and are of order 1. Thus, the transresistivity diverges logarithmically as $T \rightarrow T_c^+$ [25]. Furthermore, $\rho_{21}(T \gg T_c) \sim 1/\log(T/T_c)$, and this relatively slow falloff with increasing temperature implies that the electron-hole pairing fluctuation enhancement of ρ_{21} can be seen well above T_c . A comparison of Eq. (5a) with the weak-coupling result [19] $\rho_{21, \text{weak}} \approx -\hbar T^2 \zeta(3) \pi m_1 m_2 / (8\hbar^2 e_1 e_2 k_{F,1}^3 k_{F,2}^3 q_{TF,1} q_{TF,2} d^4)$ implies that the pairing fluctuation contribution to the transresistivity is bigger than $\rho_{21, \text{weak}}$ when

$$\log\left(\frac{T}{T_c}\right) \lesssim \frac{16\pi}{7[\zeta(3)]^2} x_1(1 - x_1) k_F^2 q_{TF,1} q_{TF,2} d^4, \quad (7)$$

where $q_{TF,i} = 2m_i e_i^2 / \kappa \hbar^2$ ($\kappa =$ dielectric constant) is the Thomas-Fermi screening length in layer i and d is the center-to-center well separation. For GaAs parameters,

with $k_F = 10^6 \text{ cm}^{-1}$ and $d = 2 \times 10^{-6} \text{ cm}$, the number on the right of Eq. (7) is approximately 300. Clearly, this result should not be interpreted quantitatively, since several approximations and assumptions have been used in this calculation. The local interlayer interaction no doubt leads to a significant overestimate of the pairing fluctuation contribution, since it fails to cut off the large momentum transfer contributions, unlike the more realistic interlayer Coulomb interaction. Furthermore, the above calculation does not take into account other factors that tend to impede electron-hole condensation. These include uncorrelated impurity potentials in the electron and hole layers (negating the time-reversal argument which makes nonmagnetic impurity scattering irrelevant in superconductivity) and band-structure effects [26]. Nevertheless, the above result strongly suggests that the enhancement can be experimentally detected.

For unmatched electron-hole densities (i.e., $\Phi \neq 0$), the density dependence of the pairing fluctuation enhanced ρ_{21} differs from the weak-coupling result. For $k_{F,1} \approx k_{F,2}$, the weak-coupling transresistivity goes as $\rho_{21, \text{weak}} \sim (k_{F,1} k_{F,2})^{-3}$; i.e., there is a monotonic increase with decrease of either density. On the other hand, since the

nesting of the electron and hole Fermi surfaces is a necessary condition for electron-hole condensation, in the pairing fluctuation enhanced case the transresistivity peaks at matched densities, $\rho_{21}(\Phi = 0) - \rho_{21}(\Phi) \propto (\Phi/T)^2$. It should be noted that a peak in the ρ_{21} at $k_{F,1} = k_{F,2}$ is also the signature of a phonon-mediated interaction. However, the phonon-mediated interaction falls rapidly ($\sim T^6$) below the Bloch-Grüneisen temperature [27] which is typically a few degrees Kelvin for GaAs, so if the peak is observed at low enough temperatures the phonon mechanism can be ruled out.

This work was initiated at Mikroelektronik Centret, Danmarks Tekniske Universitet, and partially supported by a University of Akron Summer Research Fellowship. I gratefully acknowledge useful comments from M. C. Bønsager, K. Flensberg, J. Luettmmer-Strathmann, S. M. Girvin, P. D. Gujrati, A.-P. Jauho, A. H. MacDonald, and J. W. Wilkins.

-
- [1] D. Jérôme, T. M. Rice, and W. Kohn, Phys. Rev. **158**, 462 (1967); L. V. Keldysh and Y. V. Kopayev, Fiz. Tverd. Tela **6**, 2791 (1964) [Sov. Phys. Solid State **6**, 2219 (1965)]; A. N. Kozlov and L. A. Makzimov, Zh. Eksp. Teor. Fiz. **48**, 1184 (1965) [Sov. Phys. JETP **21**, 790 (1965)]; L. V. Keldysh and A. N. Kozlov, Zh. Eksp. Teor. Fiz. **54**, 978 (1968) [Sov. Phys. JETP **27**, 521 (1968)].
- [2] Yu. E. Lozovik and V. I. Yudson, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 556 (1975) [JETP Lett. **22**, 271 (1975)]; Zh. Eksp. Teor. Fiz. **71**, 738 (1976) [Sov. Phys. JETP **44**, 389 (1976)]; S. I. Shevchenko, Fiz. Nizk. Temp. **2**, 505 (1976) [Sov. J. Low Temp. Phys. **2**, 251 (1976)].
- [3] T. Fukuzawa, E. E. Mendez, and J. M. Hong, Phys. Rev. Lett. **64**, 3066 (1990); J.-P. Cheng *et al.*, *ibid.* **74**, 450 (1995); L. V. Butov and A. I. Filin, Phys. Rev. B **58**, 1980 (1998).
- [4] G. Vignale and A. H. MacDonald, Phys. Rev. Lett. **76**, 2786 (1996).
- [5] See, e.g., X. G. Feng *et al.*, Phys. Rev. Lett. **81**, 3219 (1998), and references therein.
- [6] See, e.g., the review by A. G. Rojo, J. Phys. Condens. Matter **11**, R31 (1999), and references therein.
- [7] L. Świerkowski, J. Szymański, and Z. W. Gortel, Phys. Rev. Lett. **74**, 3245 (1995); Surf. Sci. **361/362**, 130 (1996). The authors treated correlations using a local field correction term, and found *interlayer* correlations had a negligible effect on the drag.
- [8] I. Ussishkin and A. Stern, Phys. Rev. Lett. **81**, 3932 (1998); F. Zhou and Y. B. Kim, Phys. Rev. B **59**, R7825 (1999).
- [9] M. P. Lilly *et al.*, Phys. Rev. Lett. **80**, 1714 (1998).
- [10] For a treatment of Coulomb drag with correlated disorder, see I. V. Gornyi, A. G. Yashenkin, and D. V. Khveshchenko, Phys. Rev. Lett. **83**, 152 (1999).
- [11] K. Flensberg *et al.*, Phys. Rev. B **52**, 14761 (1995).
- [12] A. Kamenev and Y. Oreg, Phys. Rev. B **52**, 7516 (1995).
- [13] R. A. Craven, G. A. Thomas, and R. D. Parks, Phys. Rev. B **7**, 157 (1973).
- [14] N. E. Bonesteel, Phys. Rev. B **48**, 11484 (1993); N. E. Bonesteel, I. A. McDonald, and C. Nayak, Phys. Rev. Lett. **77**, 3009 (1996).
- [15] J. Keller and V. Korenman, Phys. Rev. B **5**, 4367 (1972).
- [16] See, e.g., G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1990).
- [17] For simplicity, current vertex corrections are ignored; i.e., the impurities are assumed to be δ function scatterers so that the transport and lifetimes are the same. Generalization to non- δ -function impurities is not difficult (see Ref. [11]).
- [18] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Addison-Wesley, Reading, MA, 1989).
- [19] A.-P. Jauho and H. Smith, Phys. Rev. B **47**, 4420 (1993); K. Flensberg and B. Y.-K. Hu, Phys. Rev. B **52**, 14796 (1995).
- [20] U. Sivan, P. M. Solomon, and H. Shtrikman, Phys. Rev. Lett. **68**, 1196 (1992); L. Zheng and A. H. MacDonald, Phys. Rev. B **48**, 8203 (1993).
- [21] K. Yang and A. H. MacDonald, archived at cond-mat/9904061.
- [22] See, e.g., J. R. Schrieffer, *Theory of Superconductivity* (Benjamin, Reading, 1964), p. 164.
- [23] The treatment utilized here is equivalent to assuming that the local superfluid order parameter undergoes Gaussian amplitude fluctuations. The same level of approximation works very well in two-dimensional conventional superconductors so long as T_c is not approached too closely [see, e.g., M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996), 2nd ed., p. 309]. The Gaussian-amplitude-fluctuation approximation breaks down as the temperature approaches the mean-field T_c . In fact, below the mean-field T_c , the phase fluctuations dominate over the amplitude fluctuations, leading to a Kosterlitz-Thouless-type phase transition at a temperature well below the mean-field T_c [see, e.g., Nigel Goldenfeld, *Lectures on Phase Transitions and the Renormalization Group* (Addison-Wesley, Reading, 1992), p. 345]. It should be borne in mind that the calculation presented here is valid only for temperatures well above the mean-field T_c , since neither phase nor non-Gaussian amplitude fluctuations are considered.
- [24] Note that these were using the assumption that $\tau^{-1} \leq T$. For $\tau^{-1} \geq T$, roughly τ^{-1} replaces T .
- [25] It is difficult to make reliable *a priori* predictions of the magnitude of T_c due to its extreme sensitivity to the system parameters. Current estimates for GaAs under uniaxial stress (see Ref. [26]) put T_c in the 0.01–0.1 K range.
- [26] S. Conti, G. Vignale, and A. H. MacDonald, Phys. Rev. B **57**, 6846 (1998).
- [27] M. C. Bønsager *et al.*, Phys. Rev. B **57**, 7085 (1998).