To start, we will assume that the only force on the projectile is gravity.

Let's also assume that the up direction is positive.

So in the first example,

\[ \begin{align*}
v_0 & \rightarrow \text{FBD for the ball} \\
0 & \rightarrow v_x, v_y \\
v_x & \rightarrow W = mg
\end{align*} \]

The velocity vector will be tangent to the ball's path.

To find any useful information we will want to split the vector into components as follows:

\[ v_x = v_0 \cos \theta \]
\[ v_y = v_0 \sin \theta \]

Since the only force is due to gravity there is only acceleration in the vertical direction. The horizontal direction has no acceleration, so the velocity is constant.

\[ a_x = 0 \quad a_y = -g \]

Using the equations we learned in kinematics we can find some useful equations. Remembering that we want to consider the problem using components:

\[ v_x' = v_x \]
\[ x' = x + v_x t \]
\[ v_y' = v_0 \sin \theta - gt \]
\[ y' = y + v_0 \sin \theta \cdot t - \frac{1}{2} gt^2 \]
\[ (v_y')^2 = v_y^2 - 2g(y' - y) \]
It's also good to remember that if the same object is released and if the object is thrown from the same height it will take the same time to reach the ground.

Using Energy To solve Projectile Motion problems.

We can use the law of Energy conservation...

\[ K_0 + U_e = K_f + U_f \]

Energy will be conserved throughout the process.

For example...

At points 1 and 3 all the energy of the system is Kinetic.

At 1:

\[ U_1 = 0 \quad , \quad K_1 = \frac{1}{2} m (v_{x0}^2 + v_{y0}^2) \]

At 2:

\[ U_2 = mg y_2 \quad , \quad K_2 = \frac{1}{2} m v_{x0}^2 \]

\[ U_1 + K_1 = U_2 + K_2 \]

\[ \frac{1}{2} m v_{x0}^2 + \frac{1}{2} m v_{y0}^2 = mg y_2 + \frac{1}{2} m v_{x0}^2 \]

\[ \implies \frac{1}{2} m v_{y0}^2 = mg y_2 \quad \implies \quad y_2 = \frac{v_{y0}^2}{2g} \]