Reading Assignment:

- For additional information on this week's material, please review *Harmonic motion* in your Elementary Classical Physics text and read Sections 3.1 to 3.2 in Giordano and Nakanishi, *Computational Physics*.

- To prepare for next week, please review *Conservative forces* in your Elementary Classical Physics text and read Sections 3.3 to 3.4 and 3.7 in Giordano and Nakanishi, *Computational Physics*.

Exercises:

For all problems, please document your results: copy relevant figures into your word document and write figure captions that explain all symbols and include all relevant parameters. For problems 1. and 2. please upload your final programs to the dropbox.

1. The pendulum without small angle approximation

   (a) Write a Matlab program for the simple pendulum that displays the angle $\theta$ as a function of time, the angular velocity $\omega$ as a function of time, and the phase space plot, $\omega$ vs. $\theta$. Please do not use the small angle approximation for this problem, we will look at large and small angles. Please use the following variable names:
   - `theta` for the angle $\theta$
   - `omega` for the angular velocity $\omega = d\theta/dt = \dot{\theta}$
   - `omeganat` for the natural frequency $\Omega = \sqrt{g/L}$

   (b) Start by setting the initial conditions to $\theta_0 = -0.02\pi$ and $\omega_0 = 0$. Choose a final time $t_f$ of several periods. Before you run your program, determine what the graphs ($\theta(t)$, $\omega(t)$, and phase-space plot) should look like and note this in your word document. Run the program and compare the result with your expectation; resolve any discrepancies before you proceed further.

   (c) Now keep $\omega_0 = 0$, and start the pendulum at large angles, first use $\theta_0 = 0.5\pi$, then $\theta_0 = 0.999\pi$. What positions of the pendulum do these angles correspond to? (You may want to draw a sketch of the pendulum to show this.) Compare the graphs with those from (b) paying special attention to the phase-space plots.

2. Simple harmonic motion, undamped and damped

   (a) Open the files `odeharmonic.m` and `harmonic.m` in the editor and familiarize yourself with their contents. To obtain the graphs for an undamped harmonic oscillator please set the parameters and initial values to

   $$\omega_{nat} \equiv \Omega = 0.5, \quad \gamma = 0, \quad x_0 = -1, \quad v_0 = 0 \quad (1)$$
Run the program and write down the value of the total energy and the lengths of the semi axes of the ellipse in the phase space plot.

(b) On paper:
Calculate the total energy for the case
\[
\omega_{\text{nat}} \equiv \Omega = 0.5, \quad \gamma = 0, \quad x_0 = 0, \quad v_0 = 2 \cdot \Omega
\]
(2)
Is it higher or lower than in (a)? In phase space, the system should again trace out an ellipse. Calculate the values of the semi axes, write them down, and compare with (a).

(c) Change the parameters in the program to those of Eq. (2). Run the program and determine the total value of the energy and the axes of the ellipse. Do the results agree with your calculation in (b)? If not, resolve the discrepancy.

(d) Now include damping: reset the parameters \(\Omega, x_0,\) and \(v_0\) to
\[
\omega_{\text{nat}} \equiv \Omega = 0.5, \quad x_0 = -1, \quad v_0 = 0
\]
(3)
Calculate the value of \(\gamma\) that leads to critical damping. Modify the program so that you can run it four times with different damping coefficients and store the solutions. Choose damping coefficients that correspond to the over-damped, the critically damped, the under-damped, and the undamped case. Create figures (full-sized, not subplots) that show the results for the different damping coefficients for the position as a function of time, the total energy as a function of time, and the phase-space plots. Note: if the figures are too crowded, show results only for two of the damping coefficients in one graph. Compare the graphs for \(x(t)\), the total energy, and the phase-space plots, for the undamped case and the three damped cases. Please write a paragraph about similarities and differences in the graphs.