Probability

def – probability is a way of expressing knowledge or belief that an event will occur or has occurred.
* used to draw conclusions about the likelihood of potential events and the underlying mechanics of complex systems.

ex) probability of a coin
ex) Probability of dice

Heads: 50% or \( \frac{1}{2} \) 1: \( \frac{1}{2} \) 4: \( \frac{1}{6} \)
Tails: 50% or \( \frac{1}{2} \) 2: \( \frac{1}{6} \) 5: \( \frac{1}{6} \)
3: \( \frac{1}{6} \) 6: \( \frac{1}{6} \)
\( \sum \)

We denote the probability of an event \( A \) as \( P(A) \)

\( \sum P(A) \in [0, 1] \)  
* Sum of all individual probabilities is 1.

def – the complement of \( A \) is the event of \( A \) not occurring \( P(\sim A) = 1 - P(A) \)

\{ two events \}

* \( P(A \cap B) = P(A) \cdot P(B) \) \rightarrow independent events
* \( P(A \cap B) = P(A) + P(B) \) \rightarrow mutually exclusive (dependent)

\{ two events \}

* \( \frac{1}{2} \) \( \frac{1}{2} \) = \( \frac{1}{4} \)

\( \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \)
def - continuous probability distribution is a probability distribution given by a continuous cumulative distribution function.

random variables $X$, where the distribution $P(x) = 0$.

Properties

1. probability is b/w two points $P(a \leq x \leq b) = \int_a^b f(x)dx$

2. non-negative for all real $x$

3. integral of probability function is 1: $\int_{-\infty}^{\infty} f(x)dx = 1$

I. Uniform: $f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & a < x, x > b \end{cases}$

* all events of a probability distribution are equally probable

ie) coin, die
II Gaussian / Normal

* bell curve

Gaussian function: \( f(x) = e^{-x^2} \)

Gaussian Integral: \( \int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi} \)

Properties:
1. Symmetrical about mean, \( \mu \)
2. Mode = Median = \( \mu \)
3. Inflection points occur one standard deviation away from mean, \( \mu \pm \sigma \)