Abstract

The purpose of this experiment is to find coefficients of restitution of two tennis balls and compare them to the International Tennis Federation’s acceptability standards. Coefficients of restitution were calculated using data obtained from bouncing the balls repeatedly and recording their bounce heights. The tennis balls were found to have coefficients in the ITF’s acceptable range. Using Excel, the uncertainties in the measurements of the rebound heights and in the coefficients of restitution were found.

Introduction and Theory

The coefficient of restitution is a measure of how well a tennis ball bounces. It is calculated by taking the square root of the rebound height ratio:

\[ C = \sqrt{\frac{y_{\text{fin}}}{y_{\text{ini}}}} \]

Where \( y_{\text{fin}} \) is the rebound height and \( y_{\text{ini}} \) is the height from which the ball was dropped. The acceptable range for the coefficient of restitution as set by the ITF is .728 - .762.

All experimental data have uncertainties that can be determined from a statistical analysis of the data using the following equations:

\[ \text{Average Value: } \langle y \rangle = \frac{1}{N} \sum_{k=1}^{N} y_k \]

\[ \text{Standard Deviation: } \sigma^2 = \frac{1}{N-1} \sum_{k=1}^{N} (y_k - \langle y \rangle)^2 \]

\[ \sigma = +\sqrt{\sigma} \]

\[ \text{Relative Uncertainty: } \frac{\sigma}{\langle y \rangle} \times 100\% \]

Where \( N=10 \) is the number of trial bounces, \( y_k \) with \( k \) equal to 1,...,N, denotes the values of the individual measurements. The standard deviation, \( \sigma \), is a measure of how a typical measurement differs from the mean, \( \langle y \rangle \). The relative uncertainty is another way of expressing the precision of the measurements.
In this experiment, the uncertainty in the coefficient of restitution may be determined by two methods: First, by a statistical analysis of the individually calculated values for C; and second, by error propagation. From the calculations in the lab manual, the relative uncertainty of C is expected to be half as large as the relative uncertainty of the final heights:

\[
\frac{\sigma_C}{C} \approx \frac{1}{2} \frac{\sigma_{y_{\text{fin}}}}{y_{\text{fin}}} \]

The pre-factor of ½ is a consequence of the definition of C. Since a derivative of C with respect to \( y_{\text{fin}} \) was taken, the one-half exponent became the pre-factor.

**Procedure**

This experiment involved the use of two tennis balls, a 2-meter stick, and Excel software. The meter stick was placed in a stationary vertical position against a table and held by an experimenter. Another experimenter held a tennis ball at the 150cm mark of the 2-meter stick and dropped it. A third observer noted how high the ball ascended on the first bounce. The positions were recorded relative to the bottom of the ball. This procedure was repeated nine times for each ball. All of this data was recorded in Excel and used to perform calculations and create graphs.
# Data, Graphs, and Excel Calculations

<table>
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<tr>
<th>Observation</th>
<th>Rebound height</th>
<th>Rebound height</th>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>y_k (cm)</td>
<td>y_k (cm)</td>
<td>sqrt(y_k/y_ini)</td>
<td>sqrt(y_k/y_ini)</td>
</tr>
<tr>
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<td>79</td>
<td>80</td>
<td>0.7257</td>
<td>0.7303</td>
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<tr>
<td>4</td>
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<td>80</td>
<td>0.7394</td>
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<td>10</td>
<td>80</td>
<td>81</td>
<td>0.7303</td>
<td>0.7348</td>
</tr>
</tbody>
</table>

average: 80.3, 80.6, 0.73, 0.73  
standard deviation: 0.95, 0.70, 0.0043, 0.0032  
maximum: 82, 82, 0.74, 0.74  
minimum: 79, 80, 0.73, 0.73  
relative uncertainty (%): 1.18, 0.87, 0.59, 0.43

<table>
<thead>
<tr>
<th>Rebound height</th>
<th>Ball 1</th>
<th>Ball 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y(cm)</td>
<td>Counts</td>
<td>Counts</td>
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<td>1</td>
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</table>
**Calculations and Results**

The average rebound height of each ball was calculated using Eq.(2), where $N = 10$ is the number of trials and $y_k$ are the rebound heights. For ball 1 the average was 80.3 cm (see data in the Data and Graphs section). For ball 2 the average was 80.6 cm.

The standard deviation for each set of bounces was then calculated using Excel’s standard deviation function. For ball 1 this was 0.95 cm. For ball 2 this was 0.70 cm. The final results for the rebound heights were:

$$y_1 = (80.3 \pm 0.95) \text{cm}$$
$$y_2 = (80.6 \pm 0.70) \text{cm}$$

In order to construct a histogram, the minimum and maximum rebound heights for both balls are required. The maximum/minimum rebound heights were 79 cm and 82 cm for
ball 1 and 80cm and 82cm for ball 2. These data were used in setting the limits for the histogram.

Next, the relative uncertainty was calculated using Eq. (4), with the average values and standard deviation calculated above. This resulted in a relative uncertainty of 1.2% for ball 1 and .87% for ball 2.

The coefficient of restitution was calculated for each drop of each ball using Eq. (1). For example, for drop 1 of ball 1, the result was

\[
C = \frac{\sqrt{79\text{cm}}}{\sqrt{150\text{cm}}} = 0.73.
\]

The average, standard deviation, maximum and minimum values, and relative uncertainty of this data were then calculated from Eq.'s (2) through (4).

The final results for the coefficients of restitution were:

\[
C_1 = (0.73 \pm 0.0043)\text{cm}
\]

\[
C_2 = (0.73 \pm 0.0032)\text{cm}
\]

As stated in the theory section above, the standard deviation of the coefficient of restitution divided by the coefficient of restitution should be roughly equivalent to half of the standard deviation of the bounce height divided by the average final height.

For ball 1:

\[
\frac{\sigma_c}{C} = .0059
\]

\[
\frac{1}{2} \frac{\sigma_{fin}}{<y_{fin}>} = .0059
\]

For ball 2:

\[
\frac{\sigma_c}{C} = .0044
\]

\[
\frac{1}{2} \frac{\sigma_{fin}}{<y_{fin}>} = .0043
\]

**Conclusions**

The purpose of this experiment was to find the rebound height of two tennis balls and their experimental uncertainties. The results for the rebound heights were

\[
y_1 = (80.3 \pm 0.95)\text{cm}
\]

\[
y_2 = (80.6 \pm 0.70)\text{cm}
\]
We also found the coefficients of restitution for the two tennis balls and compared them to the International Tennis Federation’s acceptability standards. The results for the coefficients of restitution were

\[ C_1 = (0.73 \pm 0.0043) \text{cm} \]
\[ C_2 = (0.73 \pm 0.0032) \text{cm} \]

The results show that both balls met the ITF standards, which agrees with our perception that the balls did not seem “flat”.

**Questions**

Part 2

1) Observing the histogram, it was found that more than 2/3 of the bounce heights fell within one standard deviation for each ball.
2) In addition, 95% of the bounce heights fell within two standard deviations for each ball.

Part 3

1) The coefficient of restitution for each ball was found to be in the ITF’s allowable range.
2) The coefficients of restitution of the two balls agreed with each other within their uncertainties.
3) The data also supports the assertion that the standard deviation of the coefficient of restitution, divided by the coefficient of restitution should be roughly equal to half of the standard deviation of the bounce height divided by the average final height.
4) Some of the main sources of uncertainty in the experiment were:
   - Not having the meter stick perfectly vertical
   - The inability of the observer to judge exactly how high the ball bounced
   - The inability of the dropper to drop the ball from the same height every time
5) Some ways of avoiding these uncertainties would be:
   - Using a level to perfectly adjust the meter stick and securing it in place
   - Recording on video the ball dropping and observing the data later
   - Using a motion sensor to record the balls motion